

to the so-called high-temperature  $\theta^S$  values found in the literature. The  $\theta_{298}^S$  values obtained from the literature are identified by citations in the reference column for  $\theta_{298}^S$ .

Since the values of  $\theta_{298}^S$  are usually large when  $\theta_0^S$  values are large, and small when  $\theta_0^S$  values are small, no plot of  $\theta_{298}^S$  versus the group (similar to Fig. 21) is included here. The values of  $\theta_{298}^S$  vary from a minimum of 43°K for cesium to a maximum of 1874°K for diamond.

A plot of  $\theta_{298}^S$  for the rare earths (triangular points) is shown in Fig. 22b. It is seen that there is a small but smooth increase in  $\theta_{298}^S$  with increasing atomic number, which is approximately parallel to the Lindemann value,  $\theta^M$ . This smooth increase contrasts with the scattered data shown in Fig. 22a for  $\theta_0^S$ .

**Graphite.** Examination of  $\theta_0^S$  and  $\theta_{298}^S$  for graphite shows a very large difference ( $\theta_0^S = 402^\circ\text{K}$  and  $\theta_{298}^S = 1550^\circ\text{K}$ ). DeSorbo<sup>102</sup> has plotted  $\theta^S$  versus temperature, which shows that  $\theta^S$  increases very rapidly with temperature from  $\theta_{10}^S = 500^\circ\text{K}$  at 10°K to  $\theta_{75}^S = 900^\circ\text{K}$  at about 75°K. The values given in Table XV and the curve given by DeSorbo are in reasonable agreement.

**Estimated Data.** The estimated values of  $\theta_{298}^S$  for white and red phosphorus, scandium, technetium, protactinium, and neptunium are based on the estimated  $C_v^t$  values listed in Table XIV. For those elements for which no values are listed (monoclinic sulfur, selenium, tellurium, promethium, europium, ytterbium, polonium, francium, radium, and actinium), reasonable estimates for  $\theta_{298}^S$  may be obtained from the Lindemann equation, which is also listed in Table XV.

#### 17. LINDEMANN EQUATION AND THE DEBYE TEMPERATURE

In 1910 Lindemann<sup>103</sup> showed that the Debye temperature  $\theta^M$  is related to the melting point  $T_m$ , atomic mass  $M$ , and atomic volume  $V$ , of a substance, according to the following expression:

$$\theta^M = K(T_m/M)^{1/3}(1/V)^{1/3} \quad (17.1)$$

In Eq. (17.1)  $K$  is a constant approximately equal to 130. This expression is commonly called the Lindemann equation or the Lindemann law. Several values for  $K$  have appeared in the literature; Blackman<sup>93</sup> gives 115, Zemansky<sup>104</sup> gives 137, and Kaufman<sup>105</sup> gives 135.2. The value for  $K$  is determined by substituting experimental values of  $\theta$  in Eq. (17.1)

<sup>102</sup> W. DeSorbo, *Acta Met.* **2**, 274 (1954).

<sup>103</sup> F. A. Lindemann, *Physik. Z.* **11**, 609 (1910).

<sup>104</sup> M. W. Zemansky, "Heat and Thermodynamics," 4th ed., p. 270. McGraw-Hill, New York, 1957.

<sup>105</sup> L. Kaufman, *Trans. AIME* **224**, 1006 (1962).

for as many substances as possible. The value of  $K$  depends on the number of Debye temperatures available, since  $T_m$ ,  $M$ , and  $V$  are known for almost all the elements, and the value depends also on which Debye temperature one chooses, i.e.,  $\theta_0^S$ ,  $\theta_{298}^S$ , or some other  $\theta$ . Because this information was not given for the above literature values,  $K$  was evaluated from both  $\theta_0^S$  and  $\theta_{298}^S$ . Substituting  $\theta_0^S$  into Eq. (17.1),  $K$  was found to be  $145.1 \pm 31.6$  for 59 elements. The  $\theta_0^S$  values for graphite, gallium, germanium, ruthenium, cadmium, osmium, and iridium were not used to evaluate  $K$ . Substituting  $\theta_{298}^S$  into Eq. (17.1),  $K$  was found to be  $138.5 \pm 25.6$  for 64 elements. The  $\theta_{298}^S$  values for white and red phosphorus, rhombic sulfur, gallium, germanium, and cadmium were not used to evaluate  $K$ .

The standard deviations indicate that there is really not much difference between the two values of  $K$  obtained here. It is also seen that the range of the standard deviation for either  $K$  value includes the three literature values cited. The  $K$  value of 138.5 was used to calculate the  $\theta^M$  values listed in Table XV, primarily because its standard deviation was less than that of the  $K$  value of 145.1. The  $K$  value of 145.1 was, however, used to estimate the  $\theta_0^S$  values for those elements for which no experimental data existed.

Because of the relationship between  $\theta_0^S$  and  $\theta^M$ , and since  $\theta_0^S$  is shown in Fig. 21, no plot of  $\theta^M$  is shown for the elements of the fourth, fifth, and sixth periods of the Periodic Table. A plot, however, of  $\theta^M$  versus the atomic number for the rare-earth elements is shown in Fig. 22b. The  $\theta^M$  value increases smoothly with increasing atomic number, and with a slope similar to  $\theta_{298}^S$ . The low values for europium and ytterbium follow directly from the low melting points and the large atomic volumes of these two elements as compared with those of the other rare earths. Again this is a manifestation of the divalent character of these two elements.<sup>13</sup>

**Estimated Data.** The values for promethium and francium are shown as estimated data since their melting points and atomic volumes were both estimated. The values for radium and protactinium are shown as estimated data because the atomic volume of radium and the melting point of protactinium were estimated.

#### 18. DEBYE TEMPERATURE FROM ELASTIC CONSTANTS

Debye temperatures can be calculated from the single-crystal elastic constants  $c_{ij}$  and from the polycrystalline elastic constants  $c_l$  and  $c_t$ . The details for determining Debye temperatures are given in the reviews of Blackman,<sup>93</sup> de Launay,<sup>94</sup> Herbstein,<sup>96</sup> and Mitra,<sup>97</sup> and are not discussed here. The Debye temperatures determined from elastic constants at 0°K,  $\theta_0^S$ , and at 298°K,  $\theta_{298}^S$ , are listed in Table XVI. For convenience in making